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On Graceful Exit in String Cosmology with Pre-Big Bang Phase

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Abstract

We analyze the problem of graceful exit from superinflationary pre-big bang phase of string cosmology within the context of lowest-order string effective action. The previous no go theorems are generalized for the case when higher genus terms of general form and additional matter fields are included. It is shown that the choice of the E-frame essentially simplifies the consideration. For the example of pure gravi-dilaton case the comparison of E-frame and string frame approaches, based on phase space analysis, is carried out.

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1 Introduction

Inflationary cosmology (for reviews see [1]) was proposed as a possible solution to a number of outstanding problems of standard cosmological model such as the horizon, flatness, space-time isotropy and homogeneity, the structure formation, magnetic monopoles over-abundance problems and so on. The inflation requires a phase of accelerated expansion in the early universe and has been the subject of much investigation during the past decade. However the final form of the model is not yet fixed. The first proposed inflationary scenario, called old inflation [2], based on a first-order phase transition, could not provide a satisfactory explanation of how to get out from the inflationary phase without disturbing the good properties of the standard cosmological model [3]. The new inflationary scenario [4] was proposed to solve this graceful exit problem with second order phase transition. The field slowly rolls down the finite temperature effective potential at first, with exponential expansion occurring. Inflation terminates when the field leaves the slow rolling regime, quickly evolves to the true minimum, and reheats via oscillations about the bottom of the potential. A different solution of graceful exit problem, known as chaotic inflation was proposed in [5]. This scenario showed that inflation need not occur only in very special field theories. As in the case of new inflationary scenario, here the density fluctuations force the couplings to be excessively small and this models do suffer from a fine-tuning problems.

Recently, great interest has been devoted to the study of extensions of the inflationary scenario based on scalar-tensor theories of gravity. La and Steinhardt [6] proposed a model, known as extended inflation, based again on a first-order phase transition, where graceful exit problem was solved by using Jordan-Brans-Dicke (JBD) theory. The crucial feature of these models is that their inflationary solutions are power-law rather than exponential [7] (power-law inflationary expansion also can arise in theories of minimal gravity and an exponential scalar potential [8]). Unfortunately homogeneity afterwards is achieved only for JBD parameters which violate observations [9]. A possible way to avoid this conflict is the so-called hyperextended inflation, based on more general scalar-tensor theories [10]. The other possibility for successful extended inflation might be multidimensional theories [11]. The relationship between various theories of inflation is examined in [12].

Inflationary models in general require small parameters in particle theory Lagrangian, to provide the flat potential needed for sufficient inflation and for correct magnitude of

density fluctuations. The models of inflation with no unmotivated small parameters can be constructed by allowing for more than one field to be relevant to inflation as in "hybrid inflation" [13], soft inflation [14], and supernatural inflation [15] models.

Recently, a great deal of attention has been devoted to possible implementations of the inflationary scenario in supergravity/superstring models (see, for example, [16, 17] and references therein). The moduli fields, which parametrize perturbative flat directions of the potential in supersymmetric theories, are natural candidates to act as inflatons. An interesting alternative to the standard inflationary universe, motivated by the scale factor duality of the string effective action [18], has been developed in [19, 20, 21]. In this scenario (generically referred to as pre-big bang cosmology) the evolution starts when the string dilaton is deep in the weak coupling region and Hubble parameter is small. The evolution in this epoch is an accelerated expansion dominated by the dilaton kinetic energy and determined by the vacuum solution of the string gravi-dilaton equations of motion (kinetic inflation was also discussed in [22]). It is assumed that after a period of time, of length determined by the initial conditions, a branch change, or phase transition, from the accelerated expansion phase occurs (pre-big bang phase) into a phase which will eventually become a phase of decelerated expansion (post-big bang phase). However, confirming a previous conjecture [23] it has been shown [24] that such a branch change can not occur for a realistic dilaton potential if one is limited to the lowest order expansion of the string theory. Subsequently this result has been extended to the more general cases of gravi-dilaton-axion system with axion-dilaton potential [25] and for models with spatial curvature [26]. On the other hand, a quantum cosmological approach based on the tunnelling boundary condition results in a non-zero transition probability from a pre-big bang to a post-big bang classical solution [27]. Such a transition may be interpreted as a spatial reflection of the wavefunction in minisuperspace. In [28] it has been shown that quantum corrections arising in the strong coupling regime can regularize the curvature singularity of the tree-level pre-big bang models.

In the present paper we will discuss the graceful exit from pre-big bang phase to lowest order in the α' expansion of the string effective action for the case when higher genus terms of general form and additional matter fields are included. In the next section, the structure of the lowest order effective action is considered in both string and Einstein frames, and equations of string cosmology are derived. In Section 3 we briefly outline the pre-big bang scenario and examine the possibility of having a branch change from the accelerated expan-

sion phase into the decelerated post-big bang one. The Section 4 is devoted to phase space analysis and the relation of Einstein and string frame approaches is discussed. Finally in Section 5 we summarize our main results.

2 String Effective Action and Cosmological Equations

Perturbative string theory contains two parameters: the string tension $\alpha^{'}$ of inverse mass-squared dimension and dimensionless string coupling constant g_s . The first one sets the length scale of the theory and controls stringy effects in the sense that when $\alpha^{'} \to 0$ the theory becomes equivalent to a field theory. The second parameter g_s controls quantum effects and plays the role of loop expansion parameter. At lowest order in $\alpha^{'}$ the string effective action reads [29]

$$\begin{aligned} S &= -\frac{1}{16\pi G_D}\int d^D x\sqrt{|G|}[F_R(\varphi)R + 4F_\varphi(\varphi)\partial_M\varphi\partial^M\varphi - \frac{1}{12}F_H(\varphi)H^2 + \\ &\quad +V(\varphi) - 16\pi G_D L_m(\varphi, G_{MN}, \psi)] \end{aligned} \tag{1}$$

where R denotes the curvature scalar of the $D-$ dimensional metric G_{MN}, φ is the dilaton field, $H^2 = H_{MNP}H^{MNP}$, $H_{MNP} = 3\partial_{[P}B_{MN]}$ is the Kalb-Ramond field strength. The string coupling constant is related to the expectation value of the dilaton as $g_{s=}\langle e^{2\varphi}\rangle$. The last term in (1) denotes the Lagrangian density of other fields, collectively denoted by ψ , which is a function of the metric and dilaton field. As an example we shall consider the case of gauge field. For the heterotic string

$$L_m = \sum \frac{F_g(\varphi)}{4g_i^2}F^a_{iMN}F_i^{aMN} \tag{2}$$

where F^a_{iMN} is the gauge field strength, the index i labels the various simple components of the gauge group, while a spans the corresponding adjoint representations. The tree-level couplings g_i are given by $g_i = 1/\sqrt{k_i}$where k_i are the integer levels of the appropriate affine algebras responsible for the gauge group.

In (1) we have introduced a potential $V(\varphi)$ for the dilaton, which is expected to be non perturbative, related to supersymmetry breaking. At small couplings ($\varphi \to -\infty$) it has to go to zero as a double exponential $\exp(-\sigma\exp(-2\varphi))$, with some model dependent positive constant σ.

The functions $F_k(\varphi)$, receive string perturbative as well as non-perturbative corrections. They have the perturbative expansion

$$F_k(\varphi) = e^{-2\varphi}\left[1 + \sum Z_k^{(i)} e^{2i\varphi}\right], \tag{3}$$

where dimensionless coefficients $Z_k^{(i)}$ represent the i - loop correction. The action (1) is written in the so-called string frame metric to which test strings are directly coupled. For many purposes it is more convenient to work with Einstein frame action. Upon performing a conformal transformation given by (further we will assume that the function $F_R(\varphi)$ is positive for all values of dilaton)

$$G_{MN} = \Omega^2(\varphi)\bar{G}_{MN}, \qquad \Omega^2(\varphi) = F_R^{-2/(D-2)} \tag{4}$$

we can obtain the effective action in Einstein frame (hereafter referred to as E-frame) where the pure gravitational action takes the standard Einstein-Hilbert form:

$$\begin{aligned} S \;=\; & \int d^D x\sqrt{|G|}\{-\frac{1}{16\pi G_D}[\bar{R} + 4F\bar{G}^{MN}\partial_M\varphi\partial_N\varphi - \frac{1}{12}\bar{F}_H(\varphi)\bar{H}^2 + \\ & +\bar{V}(\varphi)] + \bar{L}_m(\varphi, \bar{G}_{MN}, \psi)\} \end{aligned} \tag{5}$$

Here the following notations are introduced (a prime denotes differentiation with respect to φ)

$$\begin{aligned} F(\varphi) &= \frac{F_\varphi}{F_R} - \frac{n}{n-1}\left(\frac{F_R'}{2F_R}\right)^2, \quad \bar{F}_H = F_H\Omega^{D-6}, \quad n = D-1 \\ \bar{V}(\varphi) &= \Omega^D V(\varphi), \quad \bar{L}_m = \Omega^D L_m(\varphi, \Omega^2\bar{G}_{MN}, \psi) \end{aligned} \tag{6}$$

If the function F_R reverses the sign at some point, then in regions with negative valued F_R the transformation similar to (4) can be performed with absolute value of function $F_R(\varphi)$. In this case the Ricci scalar enters in (5) with positive sign, which corresponds to negative sign gravitational constant. Note that at points where $F_R(\varphi) = 0$ the conformal transformation to the E-frame is singular.

By defining the conformal field ϕ as

$$\phi = \phi(\varphi), \quad \left(\frac{d\phi}{d\varphi}\right)^2 = -4\beta F, \qquad \beta = -sgnF \tag{7}$$

the pure gravi-dilaton Lagrangian density takes the form

$$L_{G\varphi} = \frac{1}{16\pi G_D}\left[-R + \beta(\partial\phi)^2 - \bar{V}\right] \tag{8}$$

In this paper we shall assume the dilaton field is non-tachionic, and therefore $\beta > 0$. At the tree-level for the functions $F_k (= e^{-2\varphi})$ we have

$$F = -\frac{1}{n-1}, \qquad \phi = \sqrt{\frac{4}{n-1}}\,\varphi \tag{9}$$

and this is indeed the case.

Let us consider D-dimensional homogeneous and isotropic metric background of Friedman - Robertson - Walker type, with time - dependent dilaton. The string frame metric is given in terms of the lapse function, $N(t)$, and scale factor $a(t)$:

$$ds^2 = N^2(t)dt^2 - a^2(t)dl^2, \tag{10}$$

where dl^2 is the metric on a n-space of constant curvature $k (= 0, \pm 1)$. Introducing this ansatze into the action (1) yields, after integrating over space and dividing by the space volume:

$$\begin{aligned} S_{eff} &= \int dt\; N a^{D-1}\{-\frac{1}{16\pi G_D}[n(n-1)F_R\left(\frac{h^2}{N^2} + \frac{f' h\dot{\varphi}}{N^2} - \frac{k}{a^2}\right) + \\ &\quad +4F\frac{\dot{\varphi}^2}{N^2} + V(\varphi)] + L\} \end{aligned} \tag{11}$$

where the dots denote time derivative, $h = \frac{\dot{a}}{a}$ is the Hubble parameter and we have introduced the notation

$$L = \frac{F_H H^2}{192\pi G_D} + L_m, \qquad f(\varphi) = \frac{2}{n-1}\ln(F_R) \tag{12}$$

The equations of motion in the $N = 1$ gauge are the following:

$$\begin{aligned} \ddot{\varphi} &= -y\dot{\varphi} - \frac{F'}{2F}\dot{\varphi}^2 + \frac{e^f}{8F}\bar{V}'(\varphi) - \frac{2\pi G_D}{F_R F}(\frac{f'}{2}T + \alpha) \\ \dot{h} &= -yh - \frac{f' b'}{2b}\dot{\varphi}^2 - k\frac{n-1}{a^2} + \end{aligned}$$

$$+\frac{e^f}{2}\left[\frac{-f^{'}}{8F}\bar{V}^{'}(\varphi)+\frac{2V}{n-1}\right]+\frac{8\pi G_D}{F_R}\varepsilon b_1 \tag{13}$$

where

$$\begin{aligned}
y &= nh+\frac{1}{2}(n-1)f^{'}\dot{\varphi}, \qquad \alpha=\frac{1}{\sqrt{|G|}}\frac{\delta\sqrt{|G|}L}{\delta\varphi} \\
T_N^M &= diag(\varepsilon,...,-p,...), \quad T_{MN}=\frac{2}{\sqrt{|G|}}\frac{\delta\sqrt{|G|}L}{\delta G^{MN}}, \\
b_1 &= \frac{1}{n}+\frac{1-np/\varepsilon}{n-1}\left(\frac{1}{n}-\frac{b^2}{4}\right)+\frac{\alpha f'}{8F\varepsilon}, \quad b^2=-\frac{n-1}{4F}f'^2
\end{aligned} \tag{14}$$

Furthermore, extremizing action (11) with respect to N yields the constraint equation:

$$\frac{16\pi G_D}{F_R}\varepsilon+e^f\bar{V}(\varphi)=n(n-1)\left[(h+f^{'}\dot{\varphi}/2)^2+k/a^2\right]+4F\dot{\varphi}^2 \tag{15}$$

As a consequence of dilaton dependence of Lagrangian L the corresponding energy - momentum tensor is acted upon by a dilaton gradient force

$$\nabla_M T_N^M=-\alpha\partial_N\varphi \tag{16}$$

where ∇_M denotes the covariant derivative defined by the string frame metric. Within the cosmological context this equation reads

$$\dot{\varepsilon}+nh(\varepsilon+p)+\alpha\dot{\varphi}=0 \tag{17}$$

In the case of vanishing potential and equation of state $p/\varepsilon = const, \alpha/\varepsilon = const$ the general solution of anisotropic multidimensional string cosmological models are considered in [19, 30, 17].

We shall now consider cosmological equations in E-frame. The associated metric is

$$ds^2=d\bar{t}^2-\bar{a}^2(\bar{t})dl^2 \tag{18}$$

According to (4) and (18) the times and scale factors in string and E-frames are related by

$$dt=e^{-f/2}d\bar{t}, \qquad a=e^{-f/2}\bar{a} \tag{19}$$

The corresponding set of cosmological equations formally can be obtained from (13), (15) by substituting $f = b = 0$ and replacing $(a, t) \to (\bar{a}, \bar{t})$:

$$\begin{aligned} \frac{d^2\varphi}{d\bar{t}^2} &= -(D-1)\bar{h}\frac{d\varphi}{d\bar{t}} - \frac{F'}{2F}\left(\frac{d\varphi}{d\bar{t}}\right)^2 + \frac{\bar{V}'(\varphi)}{8F} - \frac{2\pi G_D}{F}\alpha \\ \frac{dh}{dt} &= -nh^2 - k\frac{n-1}{a^2} + \frac{V}{n-1} + \frac{8\pi G_D}{n-1}(\varepsilon - p) \\ 16\pi G_D\varepsilon &= n(n-1)(h + k/a^2) + 4F(d\varphi/dt)^2 - \bar{V}(\varphi) \end{aligned} \tag{20}$$

If the Lagrangian density L depends only on $G_{MN}$and not on its derivatives, then E-frame energy density, pressure and function α are related to the string frame quantities through

$$\bar{\varepsilon} = \Omega^D\varepsilon, \quad \bar{p} = \Omega^D p, \quad \bar{\alpha} = \Omega^D\left(\alpha - \frac{\Omega'}{\Omega}T\right) \tag{21}$$

Introducing the new scalar function $\phi(\varphi)$ according to (7) the first of equations (20) becomes

$$\frac{d^2\phi}{d\bar{t}^2} = -n\bar{h}\frac{d\phi}{d\bar{t}} - \frac{1}{8}\bar{V}'(\varphi) - \frac{2\pi G_D}{\sqrt{\left|\bar{G}\right|}}\frac{\delta\sqrt{\left|\bar{G}\right|}L}{\delta\phi} \tag{22}$$

Assuming that the dilaton dependence of Lagrangian density L has the form

$$L = F_L(\varphi)\tilde{L}(G_{MN}, \psi) \tag{23}$$

where $\tilde{L}$ is a function of conformal weight β: $\tilde{L}(gG_{MN}, \psi) = g^\beta\tilde{L}(G_{MN}, \psi)$,it is obtained that

$$\alpha = \frac{F_L'}{F_L}L, \qquad T = -(D + 2\beta)L \tag{24}$$

Two important special cases are the Kalb - Ramond field with $\beta = -3$ and gauge field with $\beta = -2$ (see (2)). If the equation of state has the simple form $p = \lambda\varepsilon$ and $L = \lambda_0\varepsilon$ with constants λ and λ_0, the integration of (17) yields

$$\varepsilon a^{n(1+\lambda)} = const\,|F_L(\varphi)|^{-\lambda_0} \tag{25}$$

The analogous result can be also derived in E-frame.

3 Pre-Big Bang Cosmology and Graceful Exit Problem

The constraint equation (15) can be used to eliminate one of two functions h and $\dot{\varphi}$ from the cosmological equations. Here as such a variable we choose h:

$$
\begin{aligned}
h &= -f^{'}\varphi/2 \mp \\
&\mp[\left(-4F\varphi^2 + 16\pi G_D\varepsilon/F_R + e^f\bar{V}(\varphi)\right)/n(n-1) - k/a^2]^{1/2}
\end{aligned}
\tag{26}
$$

The solutions to cosmological equations belong to two branches according to which sign is chosen. First let us consider gravi-dilaton case with a flat space ($k = 0$) . By using (26) the set of cosmological equations can be written in the form of second order autonomous dynamical system with respect to variables ($\varphi, x = \dot{\varphi}$):

$$
\begin{aligned}
\dot{\varphi} &= x \\
\dot{x} &= \frac{1}{2}\left(f^{'} - \frac{F^{'}}{F}\right)x^2 \pm x\sqrt{\frac{n}{n-1}}\sqrt{-4Fx^2 + e^f\bar{V}(\varphi)} + \frac{e^f}{8F}\bar{V}^{'}(\varphi)
\end{aligned}
\tag{27}
$$

In the absence of potential the phase trajectories are defined by equation

$$
x = x_0\left(-e^f/F\right)^{1/2}\exp\left(\pm\sqrt{\frac{n}{n-1}}\phi sgnx_0\right) \tag{28}
$$

on the phase plane (φ, x), and by equation

$$
h = -\left(\frac{f^{'}}{2} \pm 2sgnx_0\sqrt{\frac{-F}{n(n-1)}}\right)x \tag{29}
$$

on the phase plane (φ, h) , with x_0 being a constant of integration. In E-frame the time dependence of this solution is given by

$$
\phi = const - sgnx_0 \cdot \sqrt{\frac{n-1}{n}}\ln|\bar{t}|\,, \quad h = \frac{1}{n\bar{t}} \tag{30}
$$

where $-\infty < \bar{t} < 0$ for the upper sign and $0 < \bar{t} < \infty$ for the lower sign. The corresponding string frame solution can be derived from the relations (28),(29) and (26). At tree-level for functions F_k ($= e^{-2\varphi}$) we obtain (see, for example, [21])

$$
2\varphi = const - (1 \pm \sqrt{n}sgnx_0)\ln|t|\,, \quad h = \mp\frac{sgnx_0}{\sqrt{n}}\frac{1}{t} \tag{31}
$$

where again $-\infty < t < 0$ for the upper sign and $0 < t < \infty$ for the lower sign. For the case $t < 0$ (upper sign in (26) and (27)) this solution describes either accelerated inflationary expansion and evolution from a flat and weakly coupled ($\varphi << -1$) universe or decelerated contraction and evolution towards weak coupling. From (27) it can be seen that for this type of solutions (following [23] we shall refer to it as (+) - branch) the minimum $\varphi = \varphi_0$ of dilaton potential $\bar{V}(\varphi)$is unstable fixed point (focus or node depending on relative values of $\bar{V}(\varphi_0)$ and $\bar{V}'(\varphi_0)$, see [31]), and therefore for such a solution dilaton cannot be fixed by potential. As it follows from (30) in E-frame the trajectories of (+) - branch describe decelerated contraction (see [19, 20] for the relation between two frames and for a discussion of their physical equivalence).

For the case $t > 0$ (lower sign in (26),(27)) the solution (31) describes either decelerated expansion or accelerated contraction depending on the sign of integration constant x_0 (see (28)). For this type of solution the minimum of potential $\bar{V}(\varphi)$ is a stable fixed point and they can be connected smoothly to a standard Friedmann-Robertson-Walker decelerated expansion with constant dilaton. In E-frame, trajectories corresponding to (–) branch solution, describe decelerated expansion.

In pre-big bang scenario of string cosmology [19, 20, 21] the pre- and post- big bang phases are realized by (+) and (–) branch solutions, correspondingly. According to this scenario the expansion of the universe starts at $t \to -\infty$ when dilaton is deep in weak coupling region ($\varphi << -1$) and the Hubble parameter is small. The evolution in this epoch (pre-big bang phase) is determined by the vacuum solution (31) of string gravi-dilaton equations with upper sign, $x_0 > 0$ and $t < 0$. After this period of superinflation, driven by dilaton kinetic energy, the universe enters into the stage where the effects of non-trivial dilaton potential and higher curvature terms become important and a branch change into a phase of decelerated expansion (post-big bang phase) occurs. In the post-big bang universe the dilaton value must be fixed, since variation of dilaton field leads to changes in masses and coupling constants, which are strongly constrained by observations. This can be realized by including dilaton potential and trapping the dilaton in a potential minimum (for another mechanism of dilaton fixation by higher genus terms see [32]). If this is the case, the post-big bang stage of evolution is described by the (–) branch solution. In this context, one of the main problems is related to the question of whether the two branches can be smoothly connected to one another. This is the graceful exit problem of the pre-big bang scenario.

To investigate the possible ways of graceful exit it is more convenient to work in E-frame rather than in string frame. In E-frame the cosmological evolution of the pre-big bang scenario looks as following. The universe contracts from the initial state when dilaton is in weak coupling region, Hubble parameter is small and scale factor is large. In this stage the evolution is determined by the (30) with $t < 0$. After some period of decelerated contraction the universe enters to the stage where non trivial dilaton potential and higher curvature terms become important. In this stage a branch change into a phase of decelerated expansion occurs. In E-frame the graceful exit problem of pre-big bang string cosmology at the classical level, corresponds to the possibility of a continuous contraction/expansion transition. It can be easily seen that such a transition cannot be simply catalyzed within the framework of the string effective action (1). Indeed, since the function $\bar{h}$ has different signs in pre- and post-big bang phases, then the continuous transition between them suggests that $\bar{h} = 0$ and $d\bar{h}/d\bar{t} > 0$ at some moment in branch changing region. But as it can be easily seen from (20)

$$\frac{d\bar{h}}{d\bar{t}} = -\frac{8\pi G_D}{n-1}(\bar{\varepsilon} + \bar{p}) - \frac{1}{n-1}\left(\frac{d\phi}{d\bar{t}}\right)^2 + \frac{k}{\bar{a}^2} \tag{32}$$

As follows from this equation $d\bar{h}/d\bar{t} < 0$ for

$$\bar{\varepsilon} + \bar{p} \geq 0, k = -1, 0 \tag{33}$$

and we conclude that branch change from pre-big bang phase to post-big bang one can not occur within the framework of lowest-order string effective action (1), if these conditions are fulfilled and

$$F_R(\varphi) > 0, F(\varphi) < 0 \tag{34}$$

as it is assumed in above analysis. Moreover as it can be easily seen, the only property of the potential $\bar{V}(\varphi)$, we have used, is its independence on metric. Therefore the previous statement on branch change impossibility is valid also for the case when the potential $\bar{V}$ depends on other scalar fields (for example,on axion field, see below).

The impossibility of (+)/(−) transition can be also seen immediately in string frame. From (26) it follows that for a continuous transition from one branch to the other it is necessary

$$\left[\left(-4F\dot{\varphi}^2 + 16\pi G_D\varepsilon/F_R + e^f\bar{V}\right)/n(n-1) - k/a^2\right]^{1/2} =$$
$$= \mp\left(h + f'\dot{\varphi}/2\right) = \mp\bar{h}e^{f/2} = 0 \tag{35}$$

where the second equation is obtained from (19). At transition point

$$\frac{d}{dt}\left(h+f'\dot{\varphi}/2\right)=e^{f/2}\frac{d\bar{h}}{d\bar{t}} \tag{36}$$

and the function $h+f'\varphi/2$ decreases for the conditions (33). Therefore, the branch change, if it takes place, must be from the (–) to the (+) branch.

By combining the last equation of (20) with (32) one finds

$$n(n-1)\frac{1}{\bar{a}}\frac{d^2\bar{a}}{d\bar{t}^2}=-8\pi G_D\left[n\bar{p}+(n-2)\bar{\varepsilon}\right]-(n-1)\left(\frac{d\phi}{d\bar{t}}\right)^2+\bar{V} \tag{37}$$

As it follows from here, if the total energy-momentum tensor, including the contribution of dilaton field, satisfies to strong energy condition, then right hand side of (37) is negative and scale factor becomes zero at some finite time moment. At contraction-expansion transition point (in E-frame) we have to have $(1/a)(d^2a/dt^2)\geq 0$, which is not the case for (37), when strong energetic condition is satisfied. In this context the no-go theorem is the consequence of Hawking-Penrose theorem on singularities [33].

Let us consider in more detail the case of Kalb-Ramond field as a source in cosmological equations. For the case of $D=4$ the equation of motion of this field

$$\partial_M\left(\sqrt{|G|}F_H(\varphi)H^{MNP}\right)=0 \tag{38}$$

can be solved by Freund-Rubin ansatz

$$H^{MNP}=\frac{1}{\sqrt{|G|}}F_H^{-1}(\varphi)\varepsilon^{MNPQ}\partial_Q A \tag{39}$$

with pseudoscalar axion field A. The corresponding contribution to the Lagrangian L (see 12) is

$$\frac{F_H^{-1}}{32\pi G_D}\partial_M A\partial^M A \tag{40}$$

which corresponds to the matter with equation of state $\varepsilon=p$, where $\varepsilon>0$ if $F_H>0$. As it follows from here the above formulated no-go theorem is valid for Kalb-Ramond field even for the case when the potential depends on axion field.

The special cases of above formulated results for $D=4$ previously have been considered

- in [23],[24]when

$$F_R=F_\varphi=e^{-2\varphi},\quad H=0,\quad L_m=0 \tag{41}$$

- [24] when
$$F_R = F_\varphi = e^{-2\varphi}, \quad H = 0 \tag{42}$$
and stringy fluid sources (in (1) L_m does not depend on dilaton field) with equation of state $p = \gamma\varepsilon, \gamma = const > -1/3$ are present.

- [25] when
$$F_R = F_\varphi = B(\varphi) \quad \mathrm{Damour-Polyakov} \quad \mathrm{ansatz} \tag{43}$$
$$H = 0, \quad L_m = 0$$

- [25],[26] when
$$F_R = F_\varphi = e^{-2\varphi}, \quad L_m = 0 \tag{44}$$
and Kalb-Ramond field is present.

4 Phase Space Analysis

We have considered the graceful exit of pre-big bang cosmology in terms of variables $(\varphi, \dot{\varphi})$. In previous investigations of this problem (see, [23, 24, 25]) usually the set of variables $(\dot{\varphi}, h)$ is chosen. Here we shall consider the relation of this approaches for the simple case of pure gravi-dilaton system with flat space ($k = 0$). First let us consider the corresponding E-frame dynamical system on phase plane $(\phi, d\phi/d\bar{t} = X)$. It can be easily obtained from (27):
$$\frac{d\phi}{d\bar{t}} = X, \quad \frac{dX}{d\bar{t}} = \pm\sqrt{\frac{n}{n-1}} X \sqrt{X^2 + \bar{V}(\phi)} - \frac{1}{2}\bar{V}'(\phi) \tag{45}$$
with Hubble parameter
$$\bar{h} = \mp\sqrt{\frac{X^2 + \bar{V}(\phi)}{n(n-1)}} \tag{46}$$
As has been previously noted the upper/lower sign corresponds to pre/post-big bang phases. For (45) classically allowed region is defined by
$$X^2 \geq -\bar{V}(\phi) \tag{47}$$
If the dilaton potential takes the negative values on some interval of dilaton field then it can be seen that the boundary of this region
$$X = \pm\sqrt{-\bar{V}(\phi)} \tag{48}$$

is a solution of system (45). As a simple example we shall consider the case of quadratic potential (the phase space analyze for the more realistic dilaton potentials, arising from gaugino condensation mechanism, see [31])

$$\bar{V}(\phi) = M^2\left(\phi^2 - \phi_1^2\right) \tag{49}$$

taking negative values on interval $-\phi_1 < \phi < \phi_1$. For $x_0\phi \to +\infty(-\infty)$the phase trajectories of dynamical system (45) with upper (lower) sign are described by equation (see (28))

$$X = x_0 \exp\left(\pm\sqrt{\frac{n}{n-1}}\phi sgn x_0\right), \quad x_0\phi \to \infty \tag{50}$$

In addition, there are special trajectories with

$$X = \pm M\sqrt{\frac{n-1}{n}} sgn\phi, \quad \phi \to \infty \tag{51}$$

For these trajectories $X^2 << \bar{V}(\phi)$ and they are potentially dominated. Note that the straight lines (51) are exact solutions of (45) if

$$\phi_1 = \phi_{10} = \sqrt{\frac{n-1}{n}} \tag{52}$$

The corresponding time dependences of E-frame scale factor and scalar field are given by

$$\bar{R} = R_0 \exp(-M^2\bar{t}^2/2n), \quad |\phi| = \pm M\sqrt{1-1/n}\bar{t}, \quad 0 \leq \pm\bar{t} < \infty \tag{53}$$

This solution is non-singular everywhere.

The qualitative structure of phase portraits of dynamical system (45) with dilaton potential (49) changes dependening on which of the following intervals the constant ϕ_1 lies:

$$(a) \quad 0 < \phi_1 < \phi_{10}, \quad (b) \quad \phi_1 = \phi_{10}, \quad (c) \quad \phi_1 > \phi_{10,} \tag{54}$$

where ϕ_{10} are defined by (52). The phase portrait corresponding to the first of this cases is presented in Fig.1. The classically forbidden region is shaded (for the potential (49) the boundary of this region is ellipse). Solid/dashed lines correspond to the E-frame expansion/contraction models (lower/upper sign in (45)). The special solutions (51) are presented by nearly horizontal trajectories (see, for example, A_1B_1). The trajectories corresponding to

the lower sign in (45) (solid lines) are attracted to the boundary (48), touche it and then the branch change into the solutions with upper sign (dashed lines) takes place. The last ones are repelled from the boundary. As we see the branch change always occurs from expansion to contraction phases. If the touching point of trajectory is the above (below) one of special trajectories with $X > 0$ ($X < 0$) then for corresponding solutions dilaton is monotonic increasing (decreasing) function of time. For trajectories touching the boundary (48) between the special trajectories (51) with $X > 0$ and $X < 0$ the time derivative $\dot{\phi}$ reverses the sign at some finite moment.

For the case (b) the touching points of special trajectories coincide with the top of boundary. These trajectories are horizontal straight lines and correspond to solutions (53). They are the only solutions with monotonic behavior of dilaton field. Finally, for the case (c) of (54) the touching points of special trajectories, coming from region with $\varphi = -\infty$ ($\varphi = +\infty$), are on positive (negative) half phase plane and there are no trajectories with monotonic dilaton field.

We shall now consider the phase trajectories of gravi-dilaton isotropic models on phase space $(\dot{\varphi}, h)$. Substituting from (15)

$$f_1\dot{\varphi} = -f'h \pm \frac{4}{\sqrt{n(n-1)}}\sqrt{-Fh^2 + \frac{1}{8}f_1 e^f \bar{V}}, \quad f_1 = \frac{8F}{n(n-1)F_R} \tag{55}$$

into (20) we obtain the following set of equations in E-frame

$$\frac{d\phi}{d\bar{t}} = \pm\sqrt{(n-1)n\bar{h}^2 - \bar{V}} \tag{56}$$

$$\frac{d\bar{h}}{d\bar{t}} = -n\bar{h}^2 + \frac{\bar{V}}{n-1} \tag{57}$$

We see that if potential is nonnegative everywhere then expansion and contraction models are separated by classically forbidden region

$$\bar{h}^2 < \bar{V}/n(n-1) \tag{58}$$

and there are no mixed expansion-contraction models. In particular, as a necessary condition for successful branch change in pre-big bang cosmology we obtain the existence of intervals with negative valued potential.

The phase portrait of dynamical system (56) for the potential (49) with (54) (a) is shown in Fig.2, where the region (58) is shaded. The solid/dashed lines correspond to the

upper/lower sign in (56). The ellipse AB_1B of Fig.1 is mapped to the interval AB with $-\phi_1 < \phi < \phi_1$. The special solutions (51) are presented by trajectories passing through points B_1 and B_2. The trajectories with initial expansion will stop expanding when h hits zero in the interval AB and h will then change the sign and the universe will begin to contract. The trajectories intersecting the ϕ axis between B_1 and B_2 present the models with monotonic dilaton. The other trajectories reach to the boundary of classically forbidden region (58) (in horizontal direction) and then reflect from it. At that point $d\phi/d\bar{t}$ reverses the sign and the trajectories described by upper (lower) sign in (56) have to turn into ones with lower (upper) sign.

To compare the analysis carried out above with previous investigations of graceful exit problem in pre-big bang string cosmology, we now turn to the case of branch change in string frame. To be consistent with previous works we should denote the trajectories having the plus/minus sign in front of square root of (55) as (+)/(−) branch trajectories. At tree level for the functions in (3) we have

$$f(\varphi) = -\frac{4\varphi}{n-1}, \quad f_1 = \frac{8}{n(n-1)} \tag{59}$$

and therefore from (55)

$$2\dot{\varphi} = nh \pm \left[nh^2 + \exp\left(-\frac{2\varphi}{n-1}\right)\bar{V}(\varphi)\right]^{1/2} \tag{60}$$

according to which the (+)/(−) branch trajectories are chosen in [23]. From (55) it follows that for a continuous (+)/(−) transition it is necessary that

$$h^2 = \frac{f_1}{8F} e^f \bar{V} \tag{61}$$

at a transition point. This equation defines the branch change curve. By using the relation of Hubble parameters in string and E-frames

$$he^{-f/2} = \bar{h} - \frac{1}{2} f'(\phi) X \tag{62}$$

we obtain the equation of branch change curve in phase plane $(\phi, \bar{h})$:

$$\bar{h} = \pm \frac{2sgnf'}{n(n-1)} \sqrt{2F\bar{V}/f_1} \equiv \pm sgnf' \bar{h}^{(0)} \tag{63}$$

In Fig.2 this curve is shown by dash-dot line for the case $f_1 > 0$ when the branch change occurs in region with negative valued potential (note that $F < 0$). At first look one would think that because the trajectories intersect the curve (63) twice then the branch change will not occur when all is said and done. However it is not difficult to see that this is not the case. By taking into account that at transition point we have to have $f_1\dot{\varphi} + f'h = 0$ from (63) one obtains

$$X = \pm\sqrt{\bar{V}\,(4/nb^2 - 1)} \equiv \pm X^{(0)} \tag{64}$$

at these points, where the upper (lower) sign corresponds to upper (lower) sign in (63) and the relation

$$f_1 = (f'^2/2)\left(1 - 4/nb^2\right) \tag{65}$$

is used. As we see, the (+)/(–) branch change occurs only when the trajectories with $X > 0$ intersect the (+) sign half of (63) or when the trajectories with $X < 0$ intersect the (–) sign half. Here we shall consider the case when the function f' never changes the sign. Since in weak coupling region $f' < 0$ (see (59)) this suggestion means that this derivative is negative for all values of dilaton field. Now we can see that in Fig.2 the trajectories in region $\bar{h} < -\bar{h}^{(0)}$ are (+) branch solutions and the trajectories in region $\bar{h} > \bar{h}^{(0)}$ are (–) branch solutions. Therefore the branch change always occurs in direction (–)→ (+) (on upper half $\bar{h} = \bar{h}^{(0)}$ for trajectories with $X < 0$ and on lower half $\bar{h} = -\bar{h}^{(0)}$ for trajectories with $X > 0$).

We now turn to the analysis of (+)/(–) branch change on phase plane (ϕ, X). In terms of these variables the (+)/(–) branch change curve is described by the equation (64). In Fig.1 this curve is shown by dash-dot line for the case $f_1 > 0$. From $X^{(0)} \geq \sqrt{-\bar{V}}$ it follows that it always lies in classically allowed region. Though the phase trajectories intersect the curve (64) twice (except the trajectories touching at points A and B) the branch change occurs only at one of these points. Namely, the branch change occurs when the trajectories with $\bar{h} < 0$ (dashed lines in Fig.1, recall that we consider the case $f' < 0$) intersect the upper half $X = X^{(0)}$ of curve (64), and then the trajectories with $\bar{h} > 0$ (solid lines) intersect the lower half $X = -X^{(0)}$. It can be easily seen that the trajectories in region $-X^{(0)} < X < -\sqrt{-\bar{V}}$ ($\sqrt{-\bar{V}} < X < X^{(0)}$) correspond to (+) ((–)) branch solutions. In the region $|X| > X^{(0)}$ the trajectories with $\bar{h} > 0$ (solid lines) correspond to (–) branch and the trajectories with $\bar{h} < 0$ (dashed lines) correspond to (+) branch solutions. We see again that branch change occurs in direction (–)→ (+).

5 Conclusions

We have considered the graceful exit problem from a superinflationary pre-big bang phase to a decelerated post-big bang one within the context of lowest order string effective action, when higher genus terms of general form and additional matter fields are present. The chosing of the E-frame essentially simplifies the consideration. We have shown that above mentioned phase transition does not occur when the conditions (33), (34) are satisfied. The branch change, if it takes place, must be always in opposite, $(-) \rightarrow (+)$ direction. As it follows from here, the possibility of successful branch change from pre-big bang phase to post big-bang one within the framework of string effective action (1) requires the violating at least of one of these conditions. Indeed, in [24] it is shown that in the case of Damour-Polyakov ansatz (43) for the functions F_k, continuous $(+) \rightarrow (-)$ branch changing solutions exist in the region there $B(\phi) < 0$. However, as it was mentioned above, in E-frame corresponding solutions are singular. In Sec.4 the phase space analysis of various approaches to the problem is carried out. The results are given in Fig.1,2.

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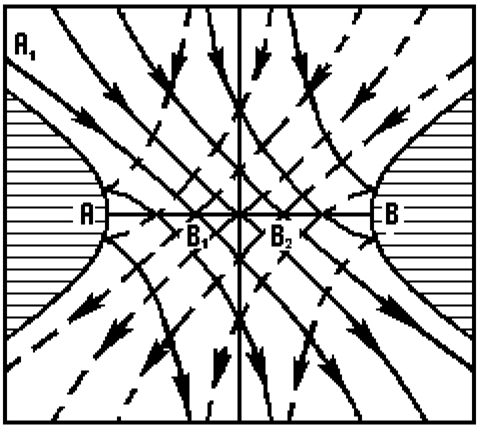
Figure Captions

Fig. 1. The phase portrait of dynamical system (45) on phase space (ϕ, X) for the potential (49) in the case (a) of (54). Solid/dashed lines correspond to the E-frame expansion/contraction models. $A_1B_1$is one of the special trajectories with asymptotic behavior (51). The dash-dot lined curve correspond to the string frame (+)/(−) branch change curve (64). The (−) → (+) branch change occur when the dashed lines intersect the upper half of this curve or when solid lines intersect the lower half.

Fig.2. The phase portrait of E-frame dynamical system (56) with potential (49) on phase space (ϕ, h) . The classically forbidden region is shaded. The trajectories passing through points B_1 and B_2 present special trajectories (51). The dash - dot line corresponds to the string frame branch changing curve (63). (−)→(+) branch change occurs when the solid line trjectories intersect lower half of this curve or when the dashed lines intersect the upper half.



A1
A
B
B1
B2